Links equations & Data set

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The code Sunfluidh solves the Navier-Stokes equations by means of an incremental projection method.

- In the prediction step, the Navier-Stokes equations are solved in order to estimate the velocity field $\vec{V}^*$ without ensuring the mass conservation ($\nabla \cdot \vec{V}=0$ for incompressible flows or $\frac{\partial \rho}{\partial t}+\nabla \cdot (\rho\vec{V})= 0$ for low Mach number flows).
- In the projection step, The mass conservation is ensured by solving a Poisson's equation. The result ϕ corresponds to the time increment of the “dynamical” pressure (the part of the pressure associated to the dynamics) and its gradient is the velocity correction that ensures the mass conservation.

$$ P_{dyn}^{n+1} = P_{dyn}^{n} + \phi$$ $$ \vec{V}^{n+1}= \vec{V}^* - \frac{\Delta t}{\rho} \nabla \phi$$

For more details on the projection methods, see the document here.

The user can read in this page :

- the different sets of governing equations that depend on the flow is either incompressible or dilatable (low Mach number hypothesis).
- the different formulations of the Poisson's equation in respect with the problem treated.
- the links between the equations, physical quantities and the data set

The governing equations

Incompressible flow formulation

Simulation of flows at moderate velocity ($ V < 0.1 \cdot Ma$) where the hypothesis $\frac{d\rho}{dt}=0$ is valid.

- Isothermal flows
- Flows with heat transfer under the Boussinesq's hypothesis
- Flows with free interface (in progress)

The equation set considered is \begin{align*} \nabla \cdot \left(\vec{V} \right) & =0 \\ \rho_0

(\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\cdot \nabla \right)\vec{V})& = -\nabla P_{dyn} + \nabla \cdot \mu \nabla(\vec{V}+^{t}\vec{V})+\vec{f}_V \\ \rho_0 C_{p0} \left(\frac{\partial T}{\partial t} + \left(\vec{V}\cdot \nabla\right)T \right)&= \nabla \cdot \lambda \nabla T \end{align*}

- ρ_0 is uniform (except in the case of flows with free interface).
- Flows with free interface are solved with a level set method (in progress)
- $\nabla \cdot \left(\vec{V} \right) =0$ is ensured by solving a Poisson's equation (see projection methods).
- When $\vec{f}_V $ is associated to the gravity/buoyancy force, it depends on the temperature variation, the thermal expansion coefficient and the gravity constant (see the Namelist "Gravity"), except for free interface flows : it also depends on the density variation.
- The dynamic viscosity and the thermal conductivity can be constant or depend on the temperature by means of the Sutherland's law (See the Namelist "Fluid_Properties").

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Low Mach number formulation

Simulation of flows at moderate velocity ($ V < 0.1 \cdot Ma$) with a noticeable variation of density (the hypothesis $\frac {d\rho}{dt}=0$ is no longer valid).

- Homogeneous gas with high temperature variation
- Multi-component gas with large density variation
- Multi-component gas with temperature variation
- Reactive flows

We remind the user that the low Mach hypothesis is based on the hypothesis of the scale splitting between the thermodynamics and dynamics phenomena. As a consequence the pressure is read as $P= P_{th}+P_{dyn}$, where P_{th} is the thermodynamical pressure and P_{dyn} the dynamical pressure. P_{th} is supposed to be uniform over the domain and is defined by the equation of state. P_dyn is solved from the Poisson's equation (see the projection methods).

The global governing equations are :

\begin{align*} \frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho\vec{V} \right) & =0 \\ \rho (\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\cdot \nabla \right)\vec{V})& = -\nabla P_{dyn} + \nabla \cdot \tau+\vec{f}_V \\ \text{where }\tau_{ij}&=\mu ((\frac{\partial V_i}{\partial x_j}+\frac{\partial V_j}{\partial x_i})-\frac{2}{3}\frac{\partial V_k}{\partial x_k}\delta_{ij}) \\ \rho

C_p \left(\frac{\partial T}{\partial t} + \left(\vec{V}\cdot \nabla\right)T \right)&= \nabla \cdot \lambda \nabla T + \frac{dP_{th}}{dt} + S_{c} + S_{r} \\ \rho \left(\frac{\partial Y_i}{\partial t} + \left(\vec{V}\cdot \nabla\right)Y_i \right)&= \nabla \cdot D_{i,m} \nabla Y_i + S_{r} + W_i \\ P_{th}&=\cdot \rho \cdot \frac{R}{M}\cdot T \end{align*}

- ρ_0 is uniform except in the case of incompressible two-phase flows.
- the mass conservation is ensured by solving a Poisson's equation (see projection methods).
- When $\vec{f}_V $ is associated to the gravity/buoyancy force, it depends on the density variation and the gravity constant (see the Namelist "Gravity").
- The dynamic viscosity, the thermal conductivity and diffusion coefficients of species can be constant, only depend on the temperature (Sutherland's law) or depend on the temeperature and species concentrations by means of relations coming from the kinetic theory of gases (See the Namelist "Fluid_Properties").

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The different formulations of the Poisson's equation

We consider

- Φ the time increment of pressure $\Phi= P_{dyn}^{n+1}- P_{dyn}^{n}$
- ρ the fluid density
- $\vec{V}^*$ the estimated velocity field solved in the prediction step
- Δt the time step
- α a time-coefficient of time scheme used for solving the Navier-Stokes equations.

The usual form for the incompressible flows is :

$$ \nabla \cdot \nabla \Phi= \frac{ \nabla \cdot \vec{V}^*}{\alpha \Delta t} $$

The form for low Mach number flows, when the density variation is moderate :

$$ \nabla \cdot \nabla \Phi= \frac{ \nabla \cdot \vec{V}^* - \frac{\partial \rho}{\partial t}}{\alpha \Delta t} $$

The form for low Mach number flows or incompressible two-phase flows, when the density variation can be large :

$$ \nabla \frac{1}{\rho}\cdot \nabla \Phi= \frac{ \nabla \cdot \vec{V}^* - \nabla \cdot \vec{V}^{n+1} }{\alpha \Delta t} $$

where $\nabla \cdot \vec{V}^{n+1}$ is estimated from the differential equation of state$

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Link between the data set & the variables in equations

List of variables	**Definition**	**namelists where the physical quantity is defined**
$\rho $	the fluid density	"Fluid_Properties", "Numerical_Methods"
$\vec{V}$	the velocity field	"Velocity_Initialization", "Velocity_Wall_Boundary_Condition_Setup", "Inlet_Boundary_Condition", "Numerical_Methods"
μ	the dynamic viscosity of the fluid	"Fluid_Properties"
P_{dyn}	the field of pressure variation related to the mass conservation (solved by a Poisson's equation – Projection method)	"Numerical_Methods"
$\vec{f}_V$	the force leading the fluid motion (gravity/buoyancy effect, external force …)	"Gravity", "External_Force"
T	the temperature field	"Fluid_Properties", "Heat_Wall_Boundary_Condition_Setup", "Inlet_Boundary_Condition", "Numerical_Methods"
P_{th}	the uniform thermodynamic pressure (low Mach number Hypothesis). Either $P_{th}=P_0$ or $P_{th}= \rho.\frac{R}{M}.T $	"Fluid_Properties" (variable “Constant_Mass_Flow)
C_p	the mass heat capacity of the fluid	"Fluid_Properties"
λ	the thermal conductivity	deduced from other data with the relation $ \lambda = \frac{\mu\cdot C_p}{Pr}$
Pr	the Prandtl number	"Fluid_Properties"
S_{c}	the chemical source term in the enthalpy equation	Namelist "Chemical_Reactions_Features"
S_{r}	the radiative source term in the enthalpy equation	in progress
Y_i	mass fraction field of the i-species	"Fluid_Properties", Namelist "Species_Properties", Namelist "Species_Initialization", Namelist "Species_Wall_Boundary_Condition_Setup", "Inlet_Boundary_Condition", "Numerical_Methods"
$D_{i,m}$	the diffusion coefficient of the species in the gas mixture	"Fluid_Properties"
W_i	chemical reaction rate	Namelist "Chemical_Reactions_Features"
R	the perfect gas constant	parameter of the code
M	the molecular mass (it is uniform if the fluid is homogeneous or depends on the species mass fractions)	"Fluid_Properties"

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